Problem:

(a) Consider a few cases:

... bab (a+b)* aba ...

aba (a+b)* bab

"bab" and "aba" are merged together,

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(b) The core recursive pattern is

\[ b^*(a^*a^*b^*a^*b^*a^*)^* \]

(c) Consider a few cases:

- Zero a

- Zero pair b adjacent a,

\[ b^* (a+b^+) \cdots (a+\varepsilon) \]

\[ b^* (a+b^+) \ast (a+\varepsilon) \]

at least one pair b adjacent a,

every pair of adjacent as precede any pair

\[(\varepsilon+b) (a^*a^* (\varepsilon+b))^* (bb (\varepsilon+\varepsilon)) (\varepsilon+b)\]
(d) "Not containing bba as suffix" means that "If bb appears then a can not follow".

Therefore, a string not containing bba consists of an initial prefix not containing bb, possibly followed by a string of bs. By including in the string of bs all the trailing bs, we may require that the initial prefix does not end with b.

Since \((a + ba)^*\) denotes the set of strings not containing bb and not ending with b, a desired regular expression is:

\[(a + ba)^* b^*\]
Problem 4.

(a) It suffices to show that for all nonnegative integers $k$, \( L^k = 1^* \) if and only if \( L^{k+1} = 1^{k+1} \).

\[ \Rightarrow \] Assume \( L^{k+1} = 1^{k+1} \).

To see that \( L^k = 1^* \), we consider:

\[
L^{k+1} = L^k \cdot 1^{k+1-1} \quad \text{by induction definition of } L
\]

\[
= L^k \cdot 1 \quad \text{by assumption}
\]

\[
= L^k \quad (\text{why}?)
\]

\[ \Leftarrow \] Assume \( L^k = 1^* \).

Let \( m \) be the length of a shortest string in \( L^k \). Then shortest string in \( L^k \cdot 1 \) and \( L^{k+1} \) are of length \( km \) and \((k+1)m\) respectively.

By assumption \( L^k = 1^* \), we should have

\[
k m = (k+1)m,
\]

that is, \( m = 0 \).

Therefore, \( \varepsilon \in L^k \).

As \( \varepsilon \in L^k \), it follows that \( 1^* \subseteq L^{k+1} \).

Hence, \( L^* = \bigcup_{i=0}^{\infty} L^i \subsetneq L^{k+1} \).

But, in addition, \( L^{k+i} \subseteq L^k \) for every \( i \geq 0 \), so, we have \( \bigcup_{i=0}^{\infty} L^i \subsetneq L^k \).

Combining \( \Rightarrow \) and \( \Leftarrow \), we have shown that

\[ L^k = L^* \iff L^{k+1} = 1^{k+1} \]
(b) The power of the given language $L$ is 3, by noting that:

- $L^2$ does not contain any string of length 5 and
- $L^3$ contains all strings of lengths $\geq 4$.

(c) Typographical error "a" should be "b", but the answer remains the same.

The power $3L$ is 00 since $3 \in L$.

(d) "Observe" that the given language $L$ contains every string in which the number of As is either "a multiple of 3" or "1 + a multiple of 3". It follows from this observation that the power $3L$ is 2.