Notes:

- Read Course Information: Section 7 (Miscellaneous) and Section 9 (Academic Dishonesty or Misconduct).
- When you are giving a construction, example, etc., provide a justification with your argument. Your solutions to numerical problems must contain the derivation of your answers. In all of your presentations, strive for correctness, completeness, and clarity. When in doubt about the assumptions of problems, the interpretations of wording, etc., consult the instructor.
- You should strive to complete all problems assigned, and a subset of them will be graded.

1. Read the notes above carefully.

2. You may need to review the prerequisite materials in discrete mathematics to have sufficient working knowledge, and then do the following exercises.

3. Provide regular expressions for following languages over the alphabet \( \Sigma = \{a, b\} \). Annotate your regular expressions or provide brief explanations for your answers — worth 50% of credits.
   
   (a) The language of all strings containing both \( bab \) and \( aba \) as substrings.
   
   (b) The set of all strings such that the number of \( a \)'s in the string is divisible by 5.
   
   (c) The set of all strings such that every pair of adjacent \( a \)'s appears before any pair of adjacent \( b \)'s.
   
   (d) The language of all strings not containing \( bba \) as a substring.

4. For a regular language \( L \), the power of \( L \) is the least (nonnegative) integer \( p \) for which \( L^p = L^p + 1 \), if such \( p \) exists, and \( \infty \) otherwise.

   (a) Consider an arbitrary regular language \( L \) over the alphabet \( \{0, 1\} \). Prove that the power of \( L \) is finite if and only if there is a nonnegative integer \( k \) such that \( L^k = L^* \), and that in this case, the power of \( L \) is the least \( k \) such that \( L^k = L^* \).

   (b) What is the power of the regular language \( \{\epsilon\} \cup \{00\}\{000\}^* \)? Justify your answer.

   (c) What is the power of the regular language \( \{a\} \cup \{00\}\{000\}^* \)? Justify your answer.

   (d) What is the power of the language denoted/represented by the regular expression \( (\epsilon + 1^*0)(1 + 01^*01^*)^* \)? Justify your answer.