Notes:

- Read Course Information: Section 7 (Miscellaneous) and Section 9 (Academic Dishonesty or Misconduct).
- When you are giving a construction, example, etc., provide a justification with your argument. Your solutions to numerical problems must contain the derivation of your answers. In all of your presentations, strive for correctness, completeness, and clarity. When in doubt about the assumptions of problems, the interpretations of wording, etc., consult the instructor.
- You should strive to complete all problems assigned, and a subset of them will be graded.

1. Read the notes above carefully.

2. For each of the following languages, prove its non-regularity by a direct application of Pumping Lemma:
   (a) \( L_1 = \{a^ib^2 \mid i \geq 1\} \).
   (b) \( L_2 = \{x \in \{a, b\}^* \mid \text{#}_a(x) \text{ is divisible by } \text{#}_b(x)\} \).

3. Consider the following languages over the alphabet \( \Sigma = \{a, b\} \):
   (a) \( L_1 = \{w \in \{a, b\}^* \mid \text{if } w \text{ contains the substring } ab, \text{ then } w \text{ contains the substring } ba\} \).
   (b) \( L_2 = \{a^ib^j c^k \mid i, j, k \geq 1, \text{ and } k \neq ij\} \).
   For each of the above languages, prove or disprove its regularity.

4. Using the closure properties of regular sets and the fact that the language \( \{a^nba^n \mid n \geq 0\} \) is not regular, prove that the language \( \{0^i1^j0^{i+j-2018} \mid i, j \geq 0, \text{ and } i + j \geq 2018\} \) is not regular.