Notes:

- Read Course Information: Section 7 (Miscellaneous) and Section 9 (Academic Dishonesty or Misconduct).
- When you are giving a construction, example, etc., provide a justification with your argument. Your solutions to numerical problems must contain the derivation of your answers. In all of your presentations, strive for correctness, completeness, and clarity. When in doubt about the assumptions of problems, the interpretations of wording, etc., consult the instructor.
- You should strive to complete all problems assigned, and a subset of them will be graded.

1. Read the notes above carefully.
4. Prove that each of the following languages is Turing-decidable (recursive):
   
   (a) $\text{DISJ}_{\text{DFA}} = \{ \langle A, B \rangle \mid A$ and $B$ are deterministic finite automata with $L(A) \cap L(B) = \emptyset \}$.
   (b) $\text{INF}_{\text{CFG}} = \{ \langle G \rangle \mid G$ is a context-free grammar such that $L(G)$ is infinite $\}$.
   (c) $\text{MIN}_{\text{DFA}} = \{ \langle A \rangle \mid A$ is a minimum-state deterministic finite automaton $\}$.
      
      Note that a minimum-state deterministic finite automaton $A$ has the minimum number of states among all deterministic finite automata accepting $L(A)$.

5. Let $\Sigma = \{0, 1\}$. Define the following language $L \subseteq \Sigma^*$:
   
   \[ L = \{ \langle M \rangle \mid M \text{ is a deterministic Turing machine that halts on at least one input string} \}. \]
   
   Prove that $L$ is Turing-recognizable (recursively enumerable).

6. Let $\Sigma = \{0, 1\}$. Assume that $A$ and $B$ are Turing-recognizable languages such that $A \cup B = \Sigma^*$. Prove that there exists a Turing-decidable language $C \subseteq \Sigma^*$ such that:
   
   \[ A \cap \overline{B} \subseteq C \text{ and } \overline{A} \cap B \subseteq C. \]